



Cambridge International AS & A Level

CANDIDATE
NAMECENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/24

Paper 2 Further Pure Mathematics 2

October/November 2025

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 It is given that $I_n = \int_0^1 x^n \sinh x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$,

$$I_n = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}. \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact value of I_2 . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



- 2 (a) Find the exact values of x for which $\cosh 2x = 6 \sinh^2 x$, giving your answers in logarithmic form. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6 \sinh^2 x$ on the same diagram. [2]





3 (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta. \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Hence obtain the roots of the equation

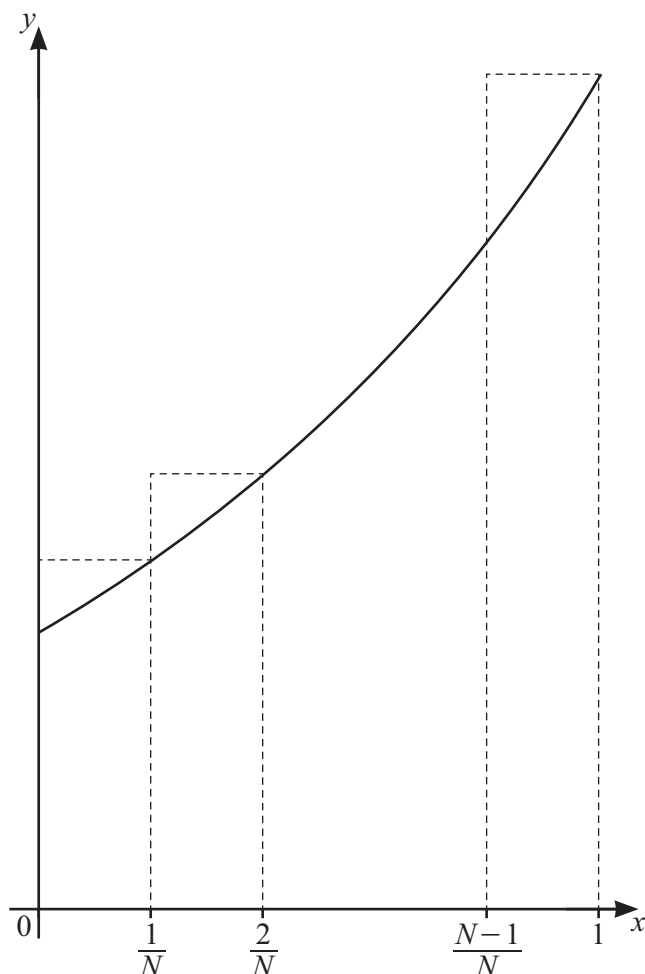
$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where q is a rational number.

[4]

[illegible]

4



The diagram shows the curve with equation $y = \frac{1}{2}(3^x)$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}(3^x) dx < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}.$$
 [4]

.....

.....

.....

.....

.....

.....

.....





- (b) Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 \frac{1}{2}(3^x) dx$. [4]

- (c) By simplifying $U_N - L_N$, show that $\lim_{N \rightarrow \infty} (U_N - L_N) = 0$. [2]





5 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x + \cos 2x. \quad [7]$$

[illegible]



- (b)** For large positive values of x and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R \sin(2x - \phi),$$

where R and ϕ are positive constants to be determined. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



6 Find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{1+x}{1-2x-x^2}y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form.

[10]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

[illegible]



7 The curve C has parametric equations

$$x = 8 \ln(\tan \frac{1}{2}t) - 3 \cot t - 3t, \quad y = 6 \ln(\tan \frac{1}{2}t) + 4 \cot t + 4t, \quad \text{for } \frac{1}{6}\pi \leq t \leq \frac{1}{3}\pi.$$

(a) (i) Show that $\frac{dx}{dt} = 8 \operatorname{cosec} t + 3 \cot^2 t$. [1]

.....

.....

.....

(ii) Find $\frac{dy}{dt}$. [1]

.....

.....

.....

(iii) Find the exact value of the length of C . [6]

[illegible]



(b) Show that

$$\frac{d^2y}{dx^2} = \frac{a \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(b \operatorname{cosec} t + c \cot^2 t)^n},$$

where a, b, c and n are integers to be determined.

[5]



- 8 (a) Find the set of values of a for which the system of equations

$$5x + ay = 3,$$

$$20x - 5y = 2,$$

$$2x - 3y + z = 1,$$

has a unique solution and interpret this situation geometrically.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Given that $a = -\frac{5}{4}$, show that the system of equations in part (a) is inconsistent and interpret this situation geometrically.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

